History of Key Technologies

Retrospective Essay on Nonlinearities in Aircraft Flight Control

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Preface and Introduction

FORTY years ago the structures of a linearized theory of servomechanisms and a linearized theory of aircraft dynamics were substantially complete. Combination and extension of the two subjects, accumulation of favorable experimental evidence, an increased general understanding, and the continuing challenge of complex and stringent requirements have made aircraft flight control a useful predictive science and a recognized engineering specialty. This much is no news to readers of the *Journal of Guidance, Control, and Dynamics*. Before we begin to congratulate ourselves, however, the editors have thought it appropriate to reflect on the perennial question from the floor: "What about nonlinearities?"

The question is often facile and impertinent. A considered response is, of necessity, complicated, extensive but not comprehensive, and invariably controversial. No general response is possible. Nevertheless, we may hope that reconsideration of specific topics will elucidate the matter and we have been persuaded to make the attempt. Our approach is historical. No new research results are presented, but our interpretation may disturb the conventional wisdom.

The earliest, more or less successful, automatic flight control systems were highly nonlinear in their sensing and actuating elements (see Fig. 1, taken from Ref. 1). Analysis, however, was at that time disdained, and performance was achieved by cut and try methods. We have told elsewhere the story of the synthesis of the art of the "tinkerer-inventor" and the science of the "theoretician" in connection with aircraft flight control.[1,2] The confluence of techniques was well developed by about 1952.[3-5] This included the experimental and theoretical study of significant nonlinearities. But the litera-

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Duane McRuer received his undergraduate and graduate education at the California Institute of Technology. Since 1957 he has been President and Technical Director of Systems Technology, Inc. His professional experience encompasses all aspects of aeronautical and astronautical control systems engineering, with primary emphasis on system development efforts concerned with manual and automatic flight control and guidance for manned and unmanned aerospace and land vehicles. In this connection he has played a responsible role in over 40 systems for transports, bombers, fighters, RPVs, missiles, research craft, and space vehicles. His major individual work has been in vehicle dynamics and stability augmentation, flying qualities, human operator dynamics, and automatic flight control and guidance. He has published over 100 technical papers and reports on these subjects, and, with Dunstan Graham, is coauthor of *Analysis of Nonlinear Control Systems* (Wiley, 1961; Dover, 1971); and *Aircraft Dynamics and Automatic Control* (Princeton University Press, 1973). Mr. McRuer has chaired and served on many government and professional society committees, including terms as President of the American Automatic Control Council (1972–1973) and Chairman of the AIAA Technical Committee on Guidance and Control (1967–1968). He is currently Chairman of the NRC's Aeronautics and Space Engineering Board and a member of the NASA Advisory Council. He has received several awards for his scientific and engineering work, including the AIAA Mechanics and Control of Flight Award, NASA's Distinguished Public Service Medal, the Franklin Institute's Levy Gold Medal, the HFS Alexander C. Williams Award, and the Caltech Distinguished Alumnus Award. He is a member of the NAE and a Fellow of the AIAA, IEEE, SAE, AAAS, and HFS.

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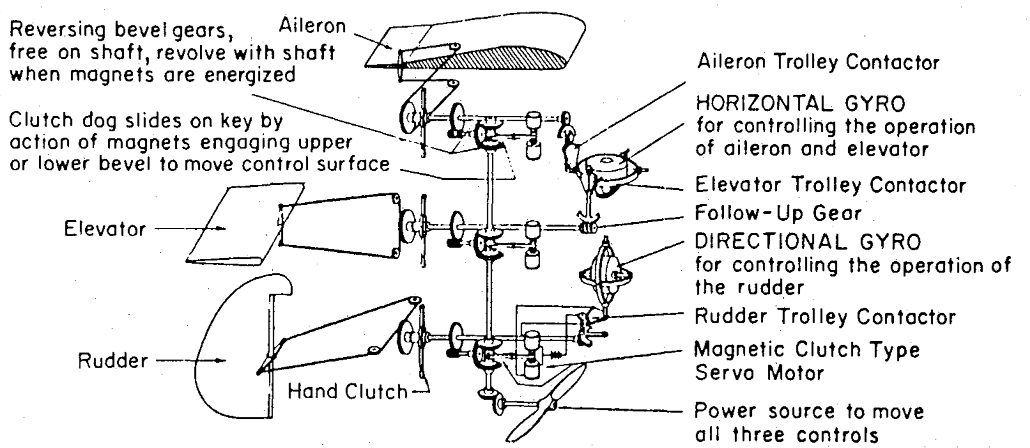


Fig. 1 Diagrammatic plan of the Sperry automatic pilot (1932). These units were ordered for Eastern Airlines' Curtiss Condor aircraft. Note the roller contact pickoffs and magnetic clutch servos.

ture of nonlinear mechanics was then very sparse. Nearly 10 years later, it was still possible for one or two individuals to have read almost everything applicable to control systems and to summarize what was useful (e.g., Ref. 6). Since then, the number of investigators and papers has grown exponentially. A large acquaintance with the literature is now quite impossible. It is, therefore, with some trepidation and appropriate modesty that we offer to the modern reader this particular and partly blinded synoptic.

The essay treats the often remarkable varieties of behavior of nonlinear systems, pervasive nonlinearities in manned aircraft and their control systems, the frustrating search for "good" nonlinearities, difficulties of analysis in nonlinear mechanics, and conclusions to be drawn from such considerations.

Remarkable Varieties of Behavior

In our terms, nonlinear systems are taken to comprise all those that are *not* "superlinear," i.e., those described by ordinary, linear, differential equations with constant coefficients, or in other words, all those to which the "transfer function method" is not appropriate. This organization of our thoughts is precise enough, but it muddles the mathematical differences between truly nonlinear systems and linear systems with varying parameters. Such distinctions, however, are only seldom useful in engineering analysis. From here on, when we say "nearly linear" we mean nearly "superlinear," and when we say strongly nonlinear, we mean the opposite. When the system is superlinear, an equilibrium is unique, and the stability of motion is global. Solutions are sums of elementary functions only. This limits the varieties of behavior in superlinear systems. On the other hand, in connection with the performance of nonlinear systems, we may discover phenomena that are *impossible* in any superlinear system. We shall attempt to treat some of these. When they are observed, they are evidence of the strongly nonlinear character of the system. When they are not observed in any finite number of analyses or tests, one cannot necessarily conclude that the system is nearly linear. This unfortunate fact is something to which we shall presently return.

For the time being, consider Table 1. There we take it that the reader will have an understanding of the mathematics and physics of the subject; exact definitions or example mathematical modes are to be found elsewhere (e.g., see Refs. 6–12). Entries in Table 1 are organized in accordance with a perception of their importance in the practice of manned aircraft flight control engineering. Thus, limit cycles and multiple equilibrium points are infrequently, but nevertheless disturbingly often enough, observed; and new frequencies are perhaps a little less troublesome. On the other hand, oscillatory jump resonance, entrainment of frequency, and chaos have not presented significant problems or opportunities in our experience of the aircraft flight control system (FCS).

Limit cycles are the usual end result of an oscillatory instability that is seldom, if ever, desired. The phenomenon commonly presents itself as a (nonlinearly) amplitude-limited oscillation at a frequency that could have been predicted by a prescient linearized analysis. When observed, they are most often evidence of a temporary failure of the design process. Redesign may eliminate the underlying linear-system condition or obviate any causative physical nonlinearity. Otherwise, it may be possible to synthetically "invert" the nonlinearity, to modify its efect by enclosing it in a high-gain feedback loop, or to "quench" the limit cycle by applying a dither signal.[13,14]

Coulomb friction and stiction, as well as signal thresholds or quantization, which all result in a minimum increment of control, will typically result in a loss of static accuracy. But these are invariably minimized by design. Other nonlinear aerodynamic or kinematic effects are often of more nearly disastrous importance in giving rise to multiple equilibria. Possible "cures" are as various as the phenomena themselves.

A nonlinear even-function characteristic of some aerodynamic forces and moments inheres in the usual plane of symmetry. The even-function variation of lift and drag with sideslip, wherein the slopes are zero only when sideslip is zero, is the most prominent example. This can couple lateral directional oscillations into longitudinal ones with half their period. Similarly, even functions appear with resolution of the gravity vector. Thus, a normal accelerometer in the presence of a roll limit cycle will exhibit twice the limit cycle frequency. These are "new" frequencies. Somewhat similar coupling may occur when the plane of symmetry is destroyed by accident or design as with oblique wing aircraft. Decoupling is a subject of continuing research interest.

The effects of amplifier saturation, control position limits, and actuator velocity limiting are the obvious examples of nonlinearities that produce a dependence of response shape (and possibly stability) on the input. Variable damping may produce similar results. Other nonexponential responses may be inescapable; they are seldom desired. Nevertheless, it is widely recognized that it is precisely in this connection that the "promise" of intentional nonlinear control systems lies.

Jump resonance, the entrainment of frequency, and chaos need to be studied by flight control engineers only enough to preclude their appearance in our systems.

More details concerning the third and fourth columns in Table 1 are presented in the next section.

Morphology and Taxonomy of Some Aircraft Control System Nonlinearities

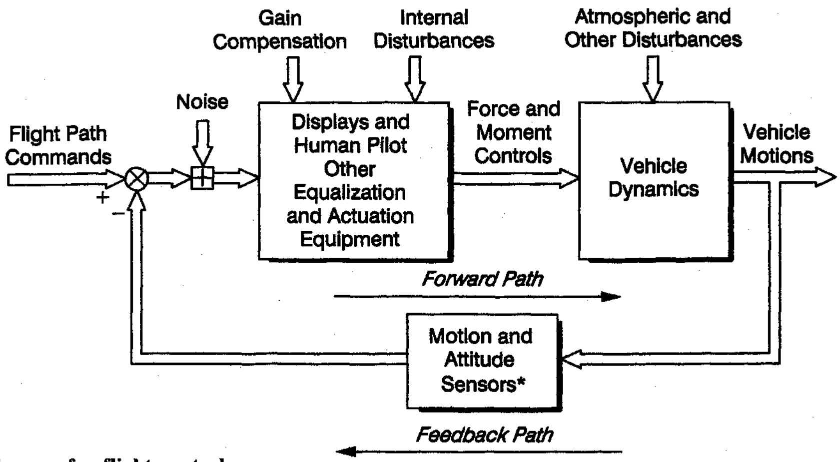
Nonlinearities are ubiquitous in dynamic systems. They are also of an infinite variety. Certainly, these statements are true in connection with flight control systems. Perhaps, fortunately, nonlinearities are often of small effect, and (with some exceptions) signals or motions that can be characterized as "small" (or at least not "large") perturbations about an equilibrium exhibit a behavior that is *nearly* linear. Use of such signals and control of such motions is then amenable to simple and effective linearized analysis and design methods. Linear concepts apply, and daunting approaches to nonlinear mechanics are avoided. When a nearly linear controller for a nearly linear controlled element is a possible solution, unambiguous answers to designer's questions concerning accuracy, stability, and dynamic response to commands and disturbances are readily available. Essentially complete "understanding" is the result. This splendid situation is much to be desired. It accounts for the fact that in the last 40 years a preponderant majority of all our aircraft flight control systems have been *designed* to be nearly linear when operating within a given system feedback architecture pertinent to a particular FCS mode. We shall later describe, as a *major and primary* aspect of nonlinear flight control, the sequencing among connected but different architectures (i.e., FCS modes) so as to accomplish command functions. Within a particular FCS mode, there remain, however, significant nonlinearities whose undesirable effects cannot be neglected or suppressed; ultimately, there is the possibility of nonlinear controllers that offer some spectacular performance advantage. Thus we may divide all significant nonlinearities into two categories: *parasitic* and *intended.* The first of these is extensive and will take the bulk of our time. Discussion of the latter (sparse) category is deferred to a later separate section.

We may now further usefully divide nonlinearities according to their place in the FCS. That is, we may consider their likely effects according to whether the system elements that exhibit them are sensors, amplifiers and signal conditioners, actuators and linkages, or the aircraft itself (see Fig. 2a). This figure will not only serve to categorize the discussion, but it will also suggest that we recall, as we proceed, some of the magic and wonder inherent in high-gain *feedback* systems. Figure 2a makes it clear that the overall FCS almost invariably *is* a feedback system; some cases of open-loop control are excepted. The figure is not explicit about the fact that many of the components and subsystems are themselves feedback control devices. Nor does it make any statements about the *profound* linearizing properties of high-gain feedback. These are, in general, well known but not yet fully explored or entirely appreciated. Figure 2b (adapted from Ref. 1) may be a helpful mnemonic. It illustrates that enclosing a nonlinearity in a high-gain feedback loop may make its effects small, perhaps negligible. This is a matter of *surpassing importance!* Associated thoughts on the influence of high-gain feedback on the effects of noise and disturbances on robustness, or the coupling between modes of motion, are also relevant to our further arguments. We merely allude to them here. Specifics may be found elsewhere (e.g., Ref. 1, pp. 511–516).

Significant nonlinearities in the sensors and their transducers are almost always very troublesome. This is because the result of any nonlinear "distortion" here appears as noise that

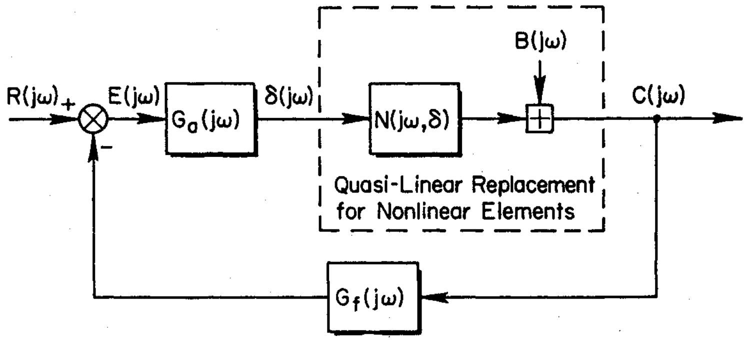
Table 1 Behavior of strongly nonlinear dynamic systems

Phenomenon	Motion	Some possible causes	Examples in aeronautics
Limit cycle	*Periodic* motion subsequent to variety of specific conditions	Various but including threshold, limiting, and hysteresis	Floating rudder with dry friction; bang-bang control; hydraulic servo with oil compressibility; possible PIO; whirl flutter; etc.
Multiple equilibria; loss of static accuracy	Possible violent transition between stable equilibria; multiple rest positions	Dry friction; nonlinear aerodynamics; kinematics	Position servos; pitch-up; rolling instability; spins
New frequencies	Frequency components in the output, *not* in the input or disturbances	Even function (rectifier) characteristic; frequency multiplier	Square law symmetric lift or drag; ankylotic motion; lateral-longitudinal couplings
Dependence of response shape and stability on input amplitude or sequence	Various	Amplifier or servo saturation; velocity limiting; variable damping	Deliberate limiting; bang-bang control; adaptive control; nonlinear yaw damper
Jump resonance	Sudden transition from one amplitude (of oscillation) to another	Hard or soft "spring"	Pitch-up is an aperiodic example
Entrainment of frequency	One oscillator slaved to another	Phase lock loop	Only in communication and navigation
Chaos	Unpredictable motion but with some "cycles," often with extreme sensitivity to specific conditions	Various	None to date; but consider Lorenz oscillator, dripping faucet problems, and weather forecasting



a) Vector block diagram for flight control

***A distinction between control and guidance often is not precise. Thus altitude, Mach number, heading, beam guidance, etc., are often included in "flight control" functions.**



b) General quasilinear high-gain feedback system block diagram

$$N(j\omega,\delta)\, G_a(j\omega)\, G_f(j\omega) \gg 1;\ \frac{C(j\omega)}{R(j\omega)} \simeq \frac{1}{G_f(j\omega)}$$

The same mathematical form of the nonlinearity has very different effects in open- and closed-loop systems, or according to whether it is in the forward or feedback path.

Fig. 2 Block diagrams.

travels around the loop with full effect and plays hob with the much desired dynamic accuracy of the system. The same is true in the early stages of signal amplification. For this reason, specialists in the design of sensors, transducers, and signal amplifiers go to great lengths to eliminate nonlinearities completely so far as this is possible. Feedback itself provides very favorable linearizing properties that help. It is often cleverly used in sensor and amplifier design. High-gain feedbacks about nonlinear elements are, in fact, historically one of the most prominent and general approaches to the alleviation of nasty nonlinearities (e.g., see Refs. 6 and 7). However, apart from absolutely unavoidable nonlinearities such as mechanical stops, the only tolerable nonlinearities in aircraft sensors, transducers, and signal amplifiers may be those whose mechanization seems desirable to reduce cost. Often this is a poor bargain that may be obviated by ingenious or serendipitous design. For example, not very long ago, many aircraft rate gyros traditionally had gimbals suspended in trunion bearings and restrained by preloaded leaf springs. This gave an undesirable threshold characteristic to the input-output relation of the instrument. Dynamic range was an important part of the specification. A more modern design has the gimbal between rotary flexures. This eliminates the nonlinearity and is more compact, lightweight, and cheaper! Many contemporary digital flight control systems derive angular rate measurements from strap-down inertial measurement units in which mechanically based threshold phenomena are essentially eliminated. But because of the quantization, the threshold/deadzone characteristics may reappear in a poorly designed system. Other modern designs use ring laser gyros. A threshold/deadzone is inherent in the physics. This is sometimes overcome by applying mechanical dither. Can this be progress?

Nonlinearities in the later stages of signal amplification and conditioning are most often intentional. As noted before, we postpone discussion of this subject.

There are parasitic nonlinearities in abundance in the aircraft's actuators and control linkages. In manual control systems, items such as stiction, coulomb friction, mechanical hysteresis, backlash, and so forth have, it seems, always been understood to be subversive of good performance.[15] Design specialists have taken much trouble to eliminate their effects. Reduction by design or "good" nonlinearities introduced to counter these noxious sources are both applicable. The usual "good" nonlinearity here is preload, which has been used to provide backup for series servo installations, to center manipulators, e.g., rudder pedals, in the presence of distributed friction, to desensitize sidesticks against inadvertent pilot inputs, to load out backlash in gears, etc. System engineers and analysts can only encourage and applaud these efforts and, at the same time, stand ready to consider the irreducible nonlinear effects. Although no doubt different in its details, the general and conventional understanding of such matters does not seem to us to have improved much for some time now.[4,15-18] Fortunately, the advent of fly-by-wire and fly-by-light flight control systems has minimized many of the number of places that

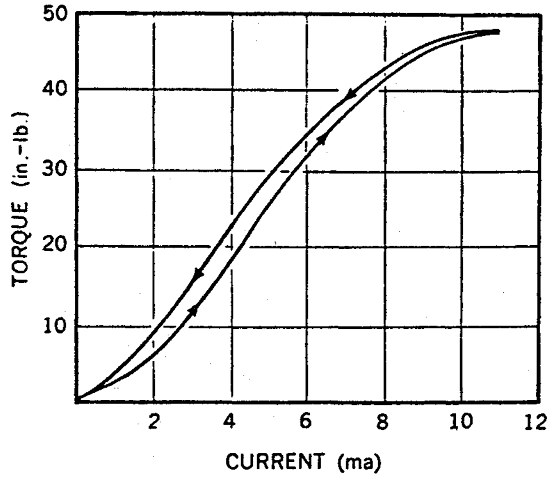


Fig. 3 Torque vs current plot for a single clutch showing hysteresis.

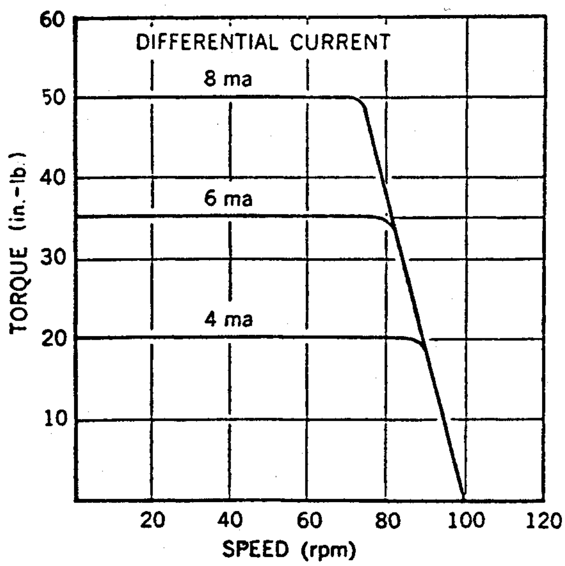


Fig. 4 Torque-speed characteristics for several differential currents.

these parasitic nonlinearities are encountered. Quite enough nonlinearities, however, sometimes remain to create major headaches in development.

Among actuators or effectors for aircraft control systems, count the human pilot and electromechanical and hydraulic (or pneumatic) servomechanisms. Each of these requires some separate consideration.

Once thought to be an *obviously* nonlinear control system element, hopelessly resistant to mathematical description, and of course not at all amenable to redesign, *mann*-in-the-loop* has come to be recognized as a quasilinear element for random-appearing tracking tasks related to piloting.[19,20] At the same time, the pilot retains spectacular nonlinear gain-changing, mode-switching, and goal-seeking precognitive control capabilities as yet only partially explored. It is also true that, to a remarkable degree, the human pilot can *invert* aircraft control system nonlinearities. In other words, he or she can supply dither, additional internal feedbacks, or just that nonlinear control function that is necessary to obviate the effects of, for example, limited detent, mechanical hysteresis, and linkage or valve friction occurring elsewhere in the manual control system. The tracking performance of the overall system is thus often nearly linear.[21] But this fortuitous result is not greatly to be depended on. The human does not like it and may give it up. Finally, in connection with pilots and nonlinearities, perhaps we should consider the causes and cures of pilot-aircraft oscillations or, more primly (and perhaps slanderously), "pilot induced oscillations" (PIO).[22-28] These are of more than one variety (e.g., "J-C" maneuvers and roll-ratchet) and can stem from several sources. By far the most common is an essentially linear phenomenon in which the pilot's gain is momentarily too high (e.g., see Refs. 22, 25, and 28). This type of PIO is exacerbated by excessive lag in the effective airplane dynamics[25] and/or by unfavorable airplane-alone quadratic dipole pairs.[22] Friction and hysteresis nonlinearities in the primary flight control system, especially when associated with bobweight-like effects, were prominent contributors to insidious and even lethal PIOs in early jet fighters. These PIOs were analogous to the limit cycle behavior of nonlinear systems. In such cases, the causes and cures lie not in the pilot but in the nonlinear characteristics of the controls. Such matters now only occasionally excite the community but often with dramatic effect.

Electromechanical servomechanisms no longer have the importance in aircraft flight control systems that they once enjoyed. In sizes now up to a few horsepower, however, they retain some remarkable advantages, and one hears the argument from time to time that they might be usefully employed in much larger sizes, such as in an all-electric aircraft. The perennial problem is to achieve lightweight power amplification, high torque to inertia ratio (bandwidth), and "sharp" null zone control. Among the more successful solutions to this problem have been clutch servomechanisms, such as in the World War II Minneapolis-Honeywell C-1 automatic pilot "pecking" servo (a wonderfully nonlinear actuator) or in the Lear contrarotating dry powder magnetic clutch servomechanisms driven by a motor already up to speed. Such a device as the latter incorporates severe parasitic nonlinearities with effects to be overcome. Taken from Ref. 29, Fig. 3 shows the variable gain and magnetic hysteresis characteristic of a single clutch. But by operating two clutches in a push-pull arrangement, each with a bias current, a "sharp" proportional null zone control is achieved. The hysteresis remains. Lastly, Fig. 4 presents the torque-speed characteristics of a complete servo. These are more reminiscent of a hydraulic than an electromechanical servo. To anyone familiar with the (more or less linear) downward sloping characteristics of reversing ac or dc servomotors that materially affect their damping in position control systems, the clutch servo characteristic would appear to be notably deficient in this respect. Now, however, damping may be supplied by high-gain feedback of a servo shaft velocity signal. Thus the servo characteristic appears as the inverse of the feedback (see Fig. 2b). It is very nearly an ideal integrator! There is no stability problem, and the hysteresis has been very nearly obviated. Position feedback was traditionally from a point near the control surface. This further reduced the effect of some control system nonlinearities. Development of this high performance, very nearly linear, aircraft servomechanism may have been the last gasp of the "tinkerer-inventors." It was carried out without benefit of either linear or nonlinear engineering analysis.

More recently (or perhaps again), aircraft servomechanisms have come to be for the most part hydraulic or (rarely) pneumatic. With regard to nonlinearities, somewhat similar considerations apply to both. The ones we choose to comment on comprise valve friction, valve flow characteristics, and velocity limiting.

Some friction in the hydraulic servo power valve is unavoidable, but it is thoroughly bad for either automatic or manual control. The difficulties are compounded by any tendency to a null zone in the torque or force characteristics of the automatic effector. Valve friction can be particularly obnoxious when the valve displacement from neutral is the physical difference between the input and output of the hydraulic actuating system (e.g., as when the valve is atop a cylinder so that a mechanical input to the valve causes a follow-up movement of

*"Mann," Anglo-Saxon for "person," is used here as a nonsexist descriptor for a human pilot.

the cylinder to close the valve). In this case, any residual valve displacement calls for an actuator velocity. Because the offset valve is followed by an effective integrator as part of the closed-loop actuator system, the describing function of the actuator system differs markedly from that of coulomb friction alone. In fact, the phase lag of this describing function can approach 180 deg at very small force inputs.[18]

Valve flow characteristics are, in fact, always nonlinear, but there seems to be some agreement that they should be as nearly linear as possible at least for small displacements. This is ordinarily accomplished by design.[30] An inevitable consequence of the desire to minimize actuator power requirements is saturation or limiting of the flow rate. In turn, this implies velocity limiting in the servomechanism. When velocity limiting occurs at low values, it is inimical to good control and can be deadly when the flight control system itself is conditionally stable (e.g., with aircraft that are statically unstable without stability augmentation). So much is established, in general, by experience and limited experiments. Just how far economy can be carried safely is hard to determine. The results are influenced by the mission segments that are tested and the disturbances encountered as well as the skill or training of the pilot. Therefore, theoretical results (e.g., Refs. 31 and 32) must be carefully related to the actual circumstances.

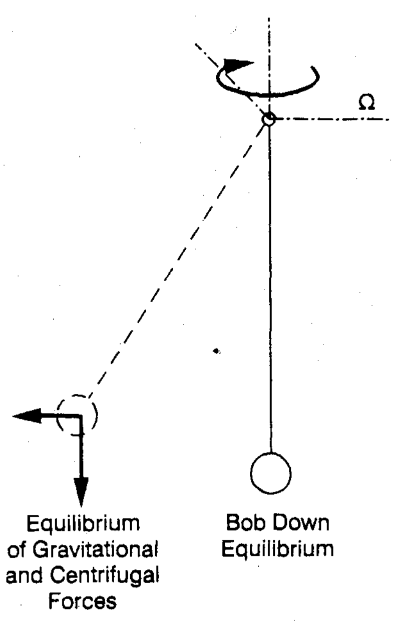
Dynamics of any airplane are, in fact, highly nonlinear and situation dependent. We do not overlook these inconveniences but are accustomed to divide and conquer with linearized means. We separate the entire flight profile into segments of established *trimmed* or "operating point" flight conditions, including accelerated equilibria, and then we consider what may be required to maintain this equilibrium (e.g., regulate against disturbances) or to introduce and maintain *small* changes in the equilibrium conditions. Gross nonlinearities inherent in the mass, mass distribution, kinematics, engine dynamics, and aerodynamics of the aircraft are reflected in the *compensation* devised for the aircraft controllers. Thus controller gains or time constants may be altered as functions of dynamic pressure q, Mach number M, trim angle of attack α, trim elevator, fuel or stores condition, turbine inlet temperature, etc. (e.g., see Ref. 1). Gain compensation is a "good" form of nonlinear control, but the community has largely lost interest in "self-adaptive" control. There still remain many particular aerodynamic and inertial nonlinearities with which an aircraft flight control engineer might find it difficult to cope. Among the "maneuvers" attributable to these are pitch up, stall departure, spin entry, and the rolling instability.[33-38]

To our way of thinking, the rolling instability is of unusual interest, since the cause and a feasible cure were revealed by theoretical analysis *before* an in-flight disaster. For 400 years, considerations of a variety of pendulums have elucidated problems in dynamics. This is still the case. Figure 5a shows a conical pendulum with a steadily rotating pivot. There is some viscous damping of the bob motion. The illustration suggests that there are at least two equilibrium positions for the bob. This then is a simple but strongly nonlinear system. We are primarily concerned with the stability of the "bob down" equilibrium. An exercise for graduate students will show that this depends on the rotational velocity of the pivot point, and the determination of stability exemplifies an elegant application of the first method of Liapounoff.[39] The conical pendulum with a rotating pivot is a very much simplified model of a rolling airplane in straight-line flight at constant speed (see Fig. 5b). There also it can be shown that the stability of perturbations in the angle of attack and sideslip depends on the rolling velocity but that the provision of a small amount of linear pitch and yaw damping will enormously expand the region of stability.[35-38]

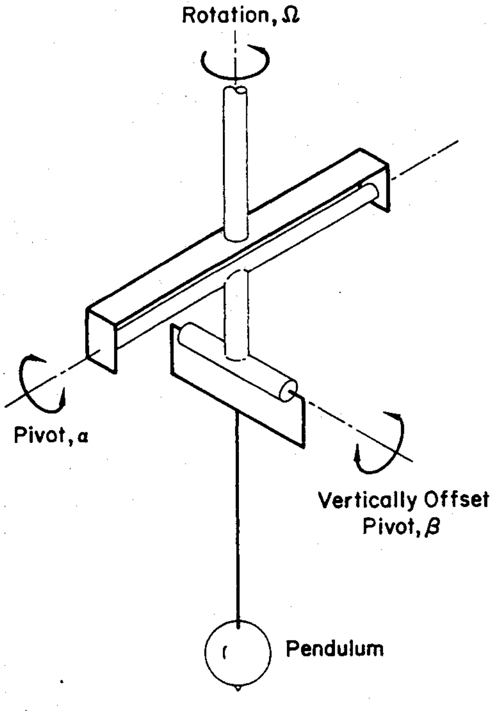
Lastly, in connection with an attempt to describe the form and to classify aircraft and control system nonlinearities, we suggest that they are further divided into two categories that may be termed simple and complex. Simple nonlinearities comprise all those with a single input and an output that is a single valued, most often antisymmetric function of the input (see Fig. 6a). These are also sometimes called "gain-changing" nonlinearities.[40] All other nonlinear functional relationships between inputs and outputs are termed complex. Figure 6b illustrates a few of these. The distinction made here, rather obviously, has to do with the possibility of general statements concerning the dynamic stability of nonlinear systems.

Is There Life in "Good" Nonlinearities?

The parasitic nonlinearities that for the most part have occupied our attention to this point are "evil" nonlinearities.



a) **Conical pendulum with rotating pivot**



b) **Rotating pendulum with offset hinge universal joint**

Fig. 5 Interesting pendulums with rotating pivot.

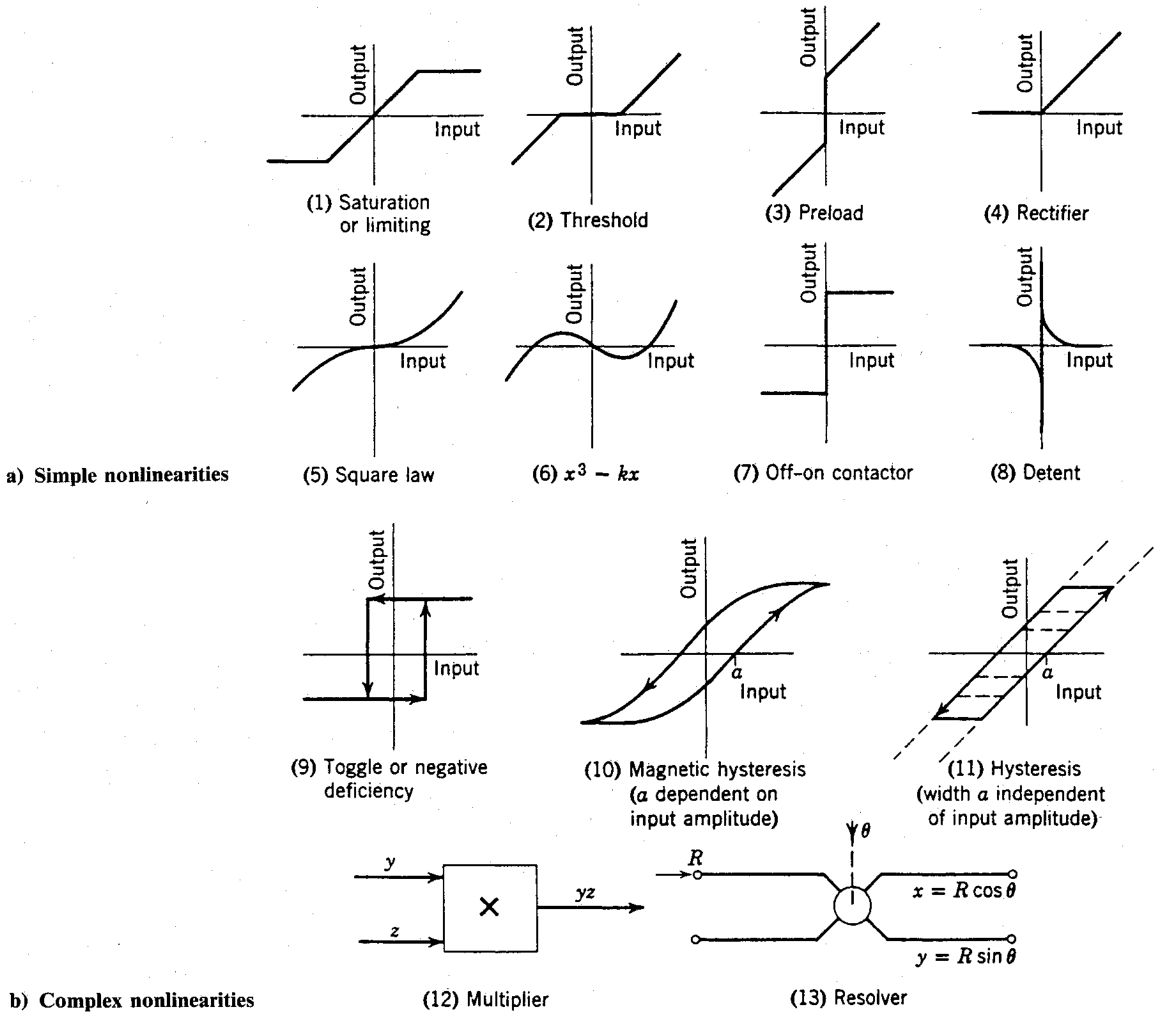


Fig. 6 Typical simple and complex nonlinearities.

Aircraft flight control engineers attempt to eliminate these nonlinearities themselves or to obviate their deleterious effects. In rather isolated specific instances, we also find intentional nonlinearities introduced to good purpose without deep thought or elegant analysis. Thus, for example, a small effective preload is retained in fly-by-wire hand controllers. Of course, this is not to mask nonexistent friction but is instead to prevent small inadvertent hand motions. Another example is the introduction of a threshold feature in load alleviation systems so that they operate only when the normal load factor is greater than some minimum value. And we have already alluded to gain compensation. This is a form of multiplicative (complex) nonlinearity not perceived as a source of peculiar behavior.

Otherwise most nonlinear features deliberately introduced into flight control systems fall conveniently into one of two categories—the nearly continuous or the essentially discrete. The nearly continuous features are typically limiting schemes of one sort or another that act in series within a given flight control system mode. They are operationally dormant except near limiting values of key airplane motion quantities. The earliest types were simple command limiters to restrict bank angle, load factor, or pitch angle commands. Limiters associated with an integrator as part of a simple feedback system are very useful elements in command rate limiting schemes. So the simple limiters, now sometimes extended to much more elaborate and sophisticated subsystems, are ubiquitous features in flight control systems to limit load factor or angle of attack or to provide protection against overspeed, tail strike, etc.

The "essentially discrete" category covers by far the most common deliberately nonlinear features in flight control. These are hardware/software combinations that provide specialized discrete command/event sequences. Redundancy management and reconfiguration functions fall into this category, although they keep score in the background and intervene only when failures are present. To the extent that the dynamics of the overall flight control system are essentially invariant in the presence of several failures, the operation of the redundancy management functions does not need to concern us here. On the other hand, "command processors," which are introduced to accomplish command and flight control mode transition operations, are definitely involved in the dynamics of normal FCS operation. These may take the form of dual mode programmed controllers. That is, in response to a command input, which ideally might be a step function, the programmed controller calls for a slewing operation wherein some aspect (e.g., rate) of the output quantity is limited until the desired final value of the command step is approached, at which point the system fades into a linear control with feedbacks appropriate for regulation of the command variable. The large amplitude slewing phase and the final linear phase constitute the "dual" mode aspects of the system. For a long time, the concept of the dual mode controller has been taken as evidence for the inchoate destiny of nonlinear control systems.[6,41] Most recently, the related concept of "sliding-mode" control[42] has attracted attention. But we retain some reservations. These are subsequently made more explicit.

In aircraft flight control, to accomplish some of the more complex commands, a sequence of several dual mode programmed controllers, with an accompanying sequence of effective autopilot feedback configurations, may be used. For example, consider a heading select command mode in which the pilot commands the autopilot to capture and maintain a new heading. In the simplest case when the heading change is very small, this is easily achieved using small bank angles. Then the direct introduction of a step heading command into the heading hold and regulation system would be all that was required, the system would be totally linear, and only the linear portion of a possible dual mode control would be in action. Capture maneuvers can be much more involved when the increment between desired and actual conditions is large. Consider the heading capture function for a more drastic heading change, when the capture maneuver itself is subjected to constraints on maximum roll rate, bank angle, etc., and smooth final capture without overshoot. Such constraints are conventionally applied in transport aircraft for the comfort and peace of mind of the passengers. The initial and final heading regulation systems will be the same as for the small heading change, but, in between, matters can be much more complicated. Because of such limiting constraints as roll-in and roll-out within a limit on rolling velocity, turning at a limited turn rate or bank angle, etc., the capture of a new heading or an instrument landing system (ILS) localizer can require a complex command processor and/or dual-mode programmed controller. The programmed controller itself is made possible by limiters, comparators, extrapolators, and threshold elements.

In spite of the nonlinear system character intrinsic to the programmed controller stages, the several effective subsystems involved in a capture sequence are themselves quite linear once they are called to duty. Informed readers can supply many more case study details from their experiences that parallel this example. The stability about transition points, which typically involve pre-transition and post-transition effective systems that are linear but different, could conceivably be a problem even when both the pre- and post-transitional states are satisfactory. There are a very limited set of forcing functions, e.g., steps or initial conditions, and for these the programmed controller features are tailored and thoroughly examined. Stability at transition in the presence of other inputs, such as suddenly applied random turbulence or noise, might be a residual issue. This is normally treated, if at all, by simulation with the final design. Experience to date shows that this has been a sufficient safeguard. In any event, the capture mode mechanizations are configuration specific, each demanding individual treatment. Therefore we cannot dwell on this topic in spite of its importance as a nonlinear feature in flight control systems. We may note, however, that one motivation for introducing the Lewis servo example, described later, is to illustrate some of the input-sensitive behavior of possible concern in programmed controller transitions. (Clever ways of accomplishing the programmed control for various flight control modes and their attendant stability at transitional points deserve separate treatment.)

Are "Good" Nonlinearities an *Ignis Fatuus?*

We turn now to possible improvements in stability or speed of response that may involve the design of a strongly nonlinear controller. The search for a nonlinear controller that promises a remarkable improvement in performance when compared to a linear design has long been a siren song for the flight control systems engineer. There is, very likely, no other control problem to which so much, mostly unrequited, effort and high talent have been devoted (recall the alchemists). By far the most encouraging examples of nonlinear aircraft control actions are exhibited by the human pilot. The quasilinear pilot regulator-like operations on random inputs can be enormously extended. Thus, by dint of preview the pilot can act as a combination open-cycle/closed-cycle controller or, with pattern recognition, develop precognitive open-loop commands or, with great skill, develop a capacity as an invertor of nonlinearities, etc.[20,21] But in all these cases the signal quantities available to the pilot and the subsequent human adjustments are not easily emulated by inanimate, automatic equipment.

Possibly the most successful results with automatic control have been obtained in connection with nonlinear features deliberately introduced to compensate for nonlinear airplane characteristics in large motions. Thus, for example, the products of angular velocities may be subtracted so as to enhance the decoupling of lateral-directional and longitudinal modes of motion in rolling maneuvers. Somewhat similarly, other possibly nonlinear crossfeeds between axes may improve matters. Otherwise, there may be some cases in which the desired command inputs are known in advance and a possibly nonlinear feedforward is introduced to improve the overall system performance in response to *that particular input.* Also, inversion procedures are becoming more and more promising as computing power and sensing capability expand.[43-45] In general, however, we may note that such ingenuity is most often suggested or inspired by an attempt to invert existing parasitic nonlinearities in flight control and regulation. In other words, we seek to make the overall performance more nearly linear! In our opinion, any list of practical nonlinear features in aircraft flight controllers is very short. And there are good reasons for this to be the case.

Anecdotal evidence, analyses, simulations, the experience of our own and others, as well as considerable "nonlinear thinking," have led us to two undemonstrated conjectures.

Conjecture 1: Given a nonlinear system with features designed to give responses to some *particular* inputs or disturbances that are superior to those of a linear system, one can, with near certainty, find a foul input or disturbance that causes a very unfortunate response.

Conjecture 2: Given an input or disturbance form that causes an unfavorable response in a strongly nonlinear aeronautical control system, there is a very high probability that it is a member of the set of all possible inputs or disturbances.

The proximate truths of these conjectures are to us the compelling explanations for the extraordinary absence of successful nonlinear control features in aeronautical flight control systems. We note that, when the input variety is confined to some much sparser set, intentional nonlinearities are often successful. This is the case in some satellite attitude control systems. We are persuaded of the validity of the conjectures on the basis of general feedback system theory and some non-aeronautical examples. With particular reference to aeronautics, it is of *surpassing* importance to consider the enormous variety in size, shape, and sequence of possible inputs and disturbances, as well as the several insertion points. (Table 10-1 on p. 540 of Ref. 1 is instructive but is only a patch on the whole subject.)

Now recall the evidence from the "Case of the Lewis Servo" (Fig. 7a). As originally conceived, this electrical, shaft-positioning servomechanism was conceived, designed, built, and tested by Professor J. B. Lewis.[46] Parenthetically, it could also be cast into the form of a simplified roll control autopilot. Lewis demonstrated, beyond doubt, that in response to the small and medium-sized step function inputs *for which it was designed* (Fig. 7b), the servo was indeed "superior" to a comparable linear device. Also, the low-frequency sinusoidal-input responses (e.g., Fig. 7c) were similar to those of a linear system. Only a little later, Caldwell and Rideout[47] discovered additional clues to its strongly nonlinear behavior by means of high-speed analog computing. They showed, for example, that for large step inputs, the response (although smooth and regular) was markedly inferior in terms of speed of response (Fig. 7b). Furthermore, with relatively high-frequency sine wave inputs, there were large error signals and frequency demultiplication (new frequencies), as shown in Fig. 7d. Caldwell and Rideout, however, failed to fully develop the ominous qualms that a more extensive investigation might have brought on (e.g., Fig. 7e). Figure 8, taken from Ref. 6, presents the results of a phase plane analysis of the Lewis servo. This diagram prominently features the never observable

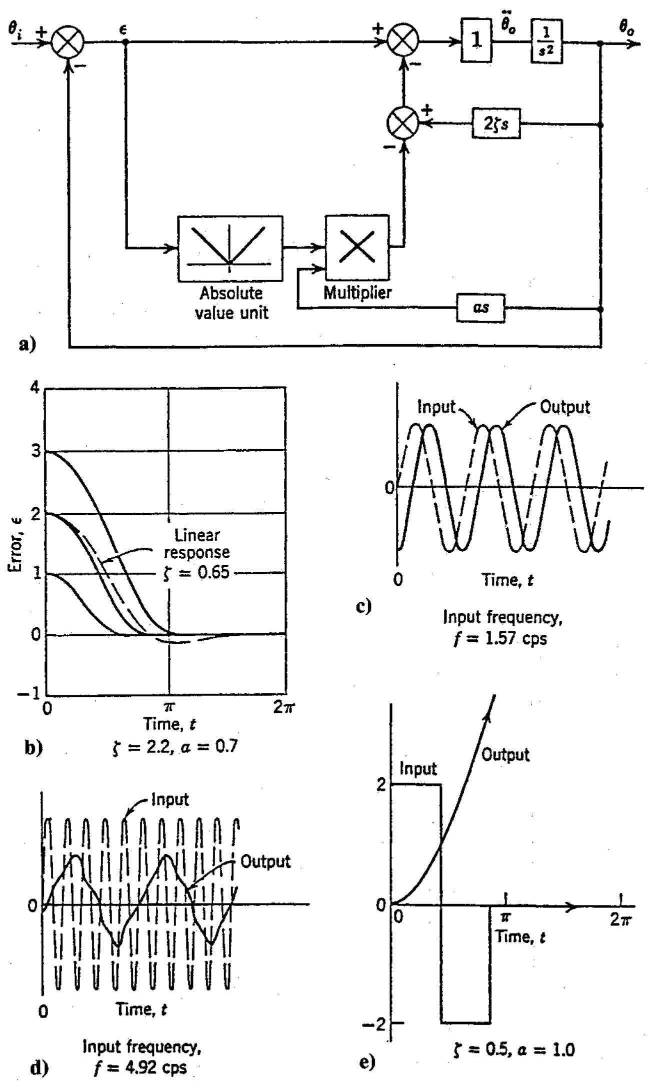


Fig. 7 Lewis servomechanism.

unstable limit cycle that bounds the region of stability for this device. It also displays the laggard response and increased percent overshoot as the input step size is increased. Only slightly reinterpreted, it suggests undesirable, increasingly oscillatory responses to larger and larger ramp inputs. Most importantly, however, it shows that under certain circumstances that may obtain before the response is complete, the addition or subtraction of another step will drive the system into *instability*, as in Fig. 7e. We learned a lot about intentional nonlinearities from that!

Difficulties and Deficiencies in Nonlinear Analysis

Analysis and synthesis of nonlinear control systems is no doubt more intellectually difficult and more severely challenging than any somewhat comparable technique based on linearized models. It is, however, a slander to allege that flight control system engineers prefer the easier transfer function and linear state space methods because such persons may appear to be somehow relatively dull or shiftless. A positive view asserts that aircraft flight control *should be* linear, and, anyway, the sum total of nonlinear mathematical methods still seems inadequate to any complete design task. In our perception, this latter parlous condition has little improved in a generation, and no significant advance may be confidently expected. Of course, there cannot be a general "Theory of Nonlinear Systems." Nonlinear mechanics is a collection of particular problems. Among the more successful, still developing, nonlinear techniques, we might list sine wave input describing functions and phase space studies (e.g., see Refs. 6-10, 14, 48-50). These tools are put to their best use in the prediction of limit cycles. But each of these has limitations in connection with aircraft flight control design. They cannot be totally relied upon. Additional techniques that are occasionally applicable are less well developed. These comprise, for example, direct solutions in terms of known functions, Gaussian random input describing functions, the second method of Liapounoff, linear inversion, stochastic nonlinear control, time scales, etc. Here, in our opinion, progress has been very slow. And it is hard to call to mind any "new math" of the last 30 years that may be of greatly significant interest to flight control systems engineers. There is a single bright spot in

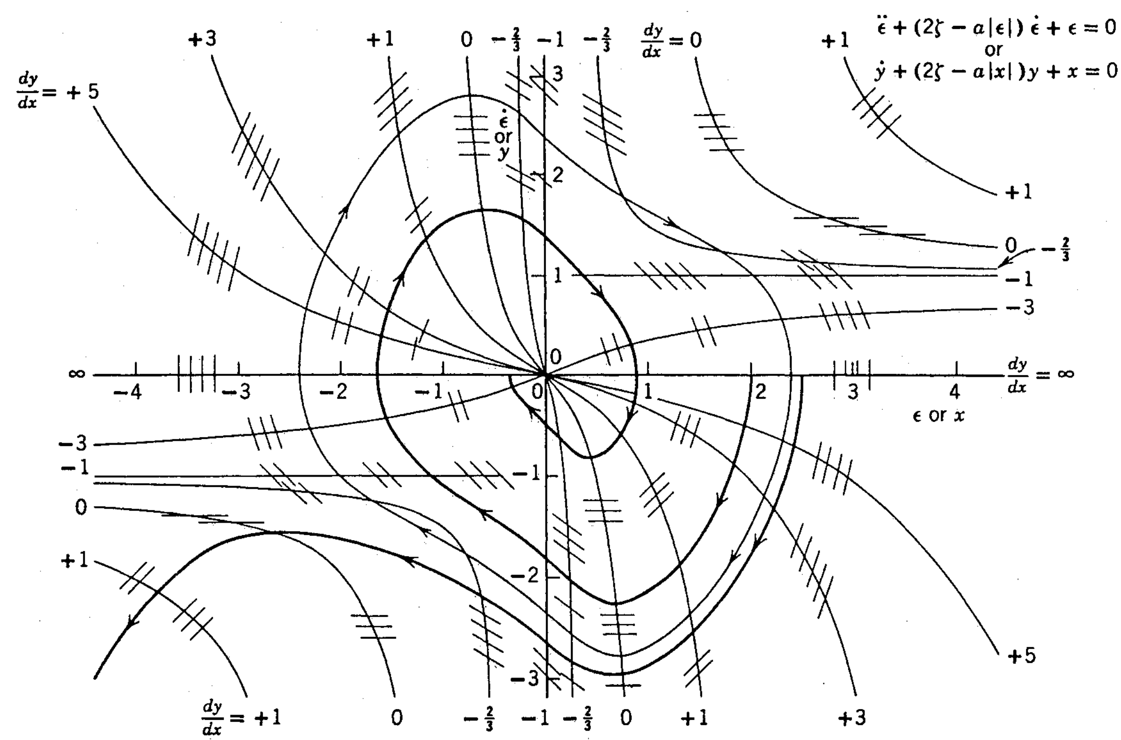


Fig. 8 Lewis servo phase plane analysis.

otherwise gloomy skies. This inheres in the possibility of numerical integration on high-speed digital computers. Today, this is the case to such an extent that, in connection with nonlinear mechanics, there is a virtually exclusive reliance on digital simulation. In our view, this is not entirely healthy. Up to his or her chin in printouts, the analyst may well be puzzled by the question, "What does it all mean?" Furthermore, the analyst *should be assailed by nagging doubts* concerning the verisimilitude of the mathematical models embodied in the program as well as artifacts of digital simulation, such as aliasing and the propagation of errors. These are seldom completely laid to rest.

On the other hand, the inherent efficiency and productivity of modern computer approaches to flight control design provide such a quantum jump in capability as to emancipate the analyst from most computational concerns once the artifacts and bugs are eliminated. This computational efficacy should perhaps change the analyst's perspective to one in which the focus is on the two conjectures. We consider these absolutely fundamental to design assessment. Conjecture 1 is the more important because nonlinear systems are almost all special cases—and finding the inputs/disturbances that can foul them is fundamental to determining any margins that the real system may exhibit.

Conclusions

"Peculiar" behavior is manifest in nonlinear mechanics. Manned aircraft and their control systems are replete with nonlinearities that lurk and bite. Designers are, most often, at great pains to eliminate nonlinearities or to obviate their effects. Except for limiters and such mechanical features as preload, the quest after "good" nonlinearities has been in large part unsuccessful. No general theory of nonlinear systems can exist, and analytical methods for particular problems are very little improved. In connection with aircraft flight control, across modes (i.e., between system architectures), envelope restrictors, and in redundancy management, discrete nonlinearities are essential. Otherwise, nonlinearities are an abomination!

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